

All in One Sample Papers

- ① Evaluate  $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$  [4m]
- ②  $\int \frac{dx}{5+4\cos x}$  or  $\int \frac{\sin 2x}{(1+\sin x)(2+\sin 2x)} dx$  [4m]
- ③ Find the area of the region bounded by the curve  $y^2 = 4ax$  &  $x^2 = 4ay$  [6m]  
 or  
 $y^2 = x$  &  $x+y = 2$
- ④  $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$  [4m]
- ⑤  $\int_0^{\pi/2} \frac{\cos x}{(\cos \frac{x}{2} + \sin \frac{x}{2})^3} dx$
- ⑥  $\int_{-1}^0 |x \cos \pi x| dx$  [6m]
- ⑦  $\int_{-1/2}^{1/2} (\cos x) \left[ \log \left| \frac{1+x}{1-x} \right| \right] dx$  [Ans 0]
- ⑧ Sketch the region  $\{(x,y) : y = \sqrt{4-x^2}\}$  and  $x$ -axis. Find the area of the region using integration. [Ans  $2\pi$ ]
- ⑨  $\int_0^1 x(\tan^{-1} x)^2 dx$  [4m] [Ans  $\frac{\pi^2}{16} - \frac{4\pi}{9} + \log \sqrt{2}$ ]
- ⑩ Area  $y^2 = 9x$  and circle  $x^2 + y^2 = 20x$  [6m]  
 [Ans  $9^2 \left( \frac{\pi}{4} - \frac{2}{3} \right)$  sq units]
- ⑪  $\int \frac{2x^2+3}{x^2+5x+6} dx$  [4m] [Ans  $2x + 11 \log|x+2| - 2 \log|x+3| + C$ ]
- ⑫  $\int_0^{\pi/2} \frac{e^{\tan^{-1} x}}{\cos^2 x + \tan^2 x} dx$  [4m] [Ans  $\frac{\pi}{4}$ ]  
 or  
 $\int_0^1 \tan^{-1} \left( \frac{1}{x^2-x+1} \right) dx$  [Ans  $\frac{\pi}{2} - \log 2$ ]
- ⑬ Area  $x^2 + y^2 = 4$  &  $(x-2)^2 + y^2 = 4$  [Ans  $\frac{8\pi}{3} - 2\sqrt{3}$ ] sq units
- ⑭  $\int \frac{dx}{\sqrt{5-4x-2x^2}}$  [4m] [Ans  $\frac{1}{\sqrt{2}} \sin^{-1} \left[ \frac{1+x}{\sqrt{7/2}} \right]$ ]  
 or  
 $\int \frac{dx}{(x+2)(x-3)}$  [Ans  $\frac{3}{5} \log|x+2| + \frac{1}{5} \log|x-3| + C$ ]

⑫  $\int \frac{x+2}{\sqrt{(x-2)(x-3)}} dx$   
 or  
 $\int \frac{\sin 2u}{(a+b\cos u)^2} du$   
 [Ans  $-\frac{2}{b^2} (\log|a+b\cos u| + \frac{a}{a+b\cos u}) + C$ ]  
 [Ans  $\frac{2}{\pi^2}$ ]

(15)  $\int_0^{\pi/4} \log(1 + \tan x) dx$  [Ans  $\frac{\pi}{8} \log 2$ ]

(16) Area of  $x^2 = 4y$  &  $x = 4y - 2$  [cm] [Ans  $\frac{2}{3}$  sq unit]

(17)  $\int_0^{\pi/4} \frac{dx}{x + \sqrt{a^2 - x^2}}$  [Ans  $\frac{\pi}{4}$ ]

(18)  $\int \frac{\sin x}{\sin 3x} dx$  [Ans  $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$ ]

✓  $\int \frac{e^x}{e^{2x} + 6e^x + 5} dx$  [Ans  $\frac{1}{4} \log \left| \frac{e^x + 1}{e^x + 5} \right| + C$ ]

(19) Find the area  $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$   
[Ans  $\frac{23}{2}$  sq units]

(20)  $\int \frac{dx}{(a^2 + x^2)^{3/2}}$  [Ans  $\frac{x}{a^2 \sqrt{a^2 + x^2}} + C$ ]

(21)  $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x} dx$  [Ans 2]

(22) Sketch the region bounded by the curves ~~and~~

$y = \sqrt{5 - x^2}$  and  $y = |x - 1|$ , then find its area. [Ans  $\frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} + \frac{5}{2}$  sq units]

Area!  $2x + y = 4, 3x - 2y = 6, x - 3y + 5 = 0$  [Ans  $\frac{7}{2}$  sq units]

(23) Find the area of the region  $\{(x, y) : y^2 \leq 4x, x^2 + y^2 \leq 9\}$   
Ans [Ans  $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{2} + \frac{\sqrt{2}}{6}$ ] sq units

$x^2 + y^2 = 1$  &  $(x - \frac{1}{2})^2 + y^2 = 1$  [Ans  $-\frac{\sqrt{15}}{8} - 2 \sin^{-1} \frac{1}{4} + \pi$ ]

(24)  $\int \frac{\sin x + \cos x}{\sqrt{9 + 16 \sin 2x}} dx$  [Ans  $\frac{1}{4} \sin^{-1} \left[ \frac{4(\sin x - \cos x)}{5} \right] + C$ ]

(25) Evaluate using limit as a sum  $\int_1^3 (2x^2 + 5x + 1) dx$

(26) Evaluate  $\int_{-1}^{3/2} |\sin \pi x| dx$  [Ans  $\frac{3}{\pi} + \frac{1}{\pi^2}$ ]

(27) Prove that the curves  $y^2 = 4x$  and  $x^2 = 4y$  divided the area of the square bounded by  $x = 0, x = 4, y = 4$  &  $y = 0$  into three equal parts.

$x + 2y = 2, y - x = 1$  &  $2x + y = 7$  [6 sq units]

(28)  $\int \frac{x^2 + x + 1}{(x^2 + 2)(x + 1)^2} dx$  [Ans  $-\frac{2}{3} \log|x+1|$  or  $\int \frac{\log x}{(1 + \log x)^2} dx$  [Ans  $\frac{x}{1 + \log x}$ ]]

① Find  $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta$  Ans  $3 \log |\sin \theta - 2| - \frac{4}{\sin \theta - 2} + c$

or  $\int_0^{\pi} e^{2x} \sin\left(\frac{\pi}{4} + x\right) dx$  Ans  $\frac{1}{5\sqrt{2}} [e^{2\pi} + 1]$

②  $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$  Ans  $\frac{2}{3} \sin^{-1} \sqrt{\frac{x^3}{a^3}}$

③  $\int_{-1}^2 |x^2 - x| dx$  Ans  $\frac{11}{4}$

④ Find the area of the region

$\{(x, y) : x^2 + y^2 \leq 2ax, y^2 \geq ax, x, y \geq 0\}$

= X =

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①  $\int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx$  Ans =  $\frac{e^{2x} (2x-3)^{-2}}{2} + c$

or  $\int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} dx$  Ans =  $\frac{1}{5} \left[ \log |(x^2 + 1)(x + 2)^3| + \tan^{-1} x \right] + c$

②  $\int_{-2}^2 \frac{x^2}{1 + 5^x} dx$  Ans  $\frac{8}{3}$

③  $\int (x+3) \sqrt{3-4x-x^2} dx$  Ans =  $\frac{(3-4x-x^2)^{3/2}}{3} + \frac{x+2}{2} \sqrt{3-4x-x^2} + \frac{1}{\sqrt{7}} \sin^{-1} \left( \frac{x+2}{\sqrt{7}} \right) + c$

④ Find the area of triangular region (2, -2) (4, 3) (1, 2)  
Ans  $\frac{13}{2}$  unit

= X =

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①  $\int \frac{dx}{5-8x-x^2} dx$  Ans  $\frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21+x+4}}{\sqrt{21-x-4}} \right| + c$

②  $\int \frac{\cos \theta}{(4 + \sin^2 \theta)(\sqrt{5} - 4 \cos^2 \theta)} d\theta$  Ans  $-\frac{1}{80} \tan^{-1} \left( \frac{\sin \theta}{2} \right) + \frac{2}{15} \tan^{-1} (2 \sin \theta) + c$

③  $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$  Ans  $\frac{\pi}{2} (\pi - 2)$

or

④  $\int_1^4 \{ |x-1| + |x-2| + |x-4| \} dx$  Ans  $\frac{23}{2}$

(4) Area of triangle with vertices (4,1), (6,6), (8,4) [Ans 7 sq units]

or

$$4y = 3x^2 \quad \& \quad 3x - 2y + 12 = 0 \quad [27 \text{ sq units}]$$

              

Set II & III

Ans  $\frac{1}{6} \tan^{-1} \frac{\cos \theta}{2} - \frac{1}{3} \tan^{-1} \cos \theta + C$

(1)  $\int \frac{\sin \theta d\theta}{(4 + \cos^2 \theta)(2 - \sin^2 \theta)}$

(2)  $\int \frac{e^x dx}{(e^x - 1)^2 (e^x + 2)}$       Ans  $\frac{1}{3} \log \left| \frac{e^x + 2}{e^x - 1} \right| - \frac{1}{3(e^x - 1)} + C$

~~$\int_2^{\infty} \frac{2^x dx}{x}$~~

(3)  $\int \frac{dx}{x^2 + 4x + 8}$       Ans  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x+2}{\sqrt{2}} \right) + C$

(4)  $\int \frac{2x dx}{(x^2 + 1)(x^2 + 2)^2}$       Delhi Set I, II & III  
 Ans  $\log \left( \frac{x^2 + 1}{x^2 + 2} \right) + \frac{1}{x^2 + 2} + C$

(5)  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$        $\left[ \frac{\pi^2}{4} \right]$

(6)  $\int_0^{3/2} |x \sin \pi x| dx$        $\left[ \frac{2}{\pi} + \frac{1}{\pi^2} \right]$

(6) Area (-2,1), (0,4), (2,3) [4 sq units]

or

$$x^2 + y^2 = 16 \quad \& \quad \sqrt{3} y = x \quad \left[ 4 \frac{\pi}{3} \right]$$

$\int \frac{2x dx}{(x^2 + 1)(x^4 + 4)}$

(7)  $\int \frac{(3 \sin x - 2) dx}{13 - \cos^2 x - 7 \sin x}$       Ans  $10 \log |\sin x - 4| - 7 \log |\sin x - 2| + C$