

1-6	7-12	13-22	23-30
1	2	3	4

Trigonometry

- ① If $\sin \theta + \cos \theta = 1$, then prove that $\tan \theta + \cot \theta = 1$ [3m]
- ② If $\tan A = n \tan B$ and $\sin A = m \sin B$, then prove that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$ [4m]
- ③ Prove that $\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$ [2m]
- ④ Prove that $(\tan A + \operatorname{cosec} B)^2 - (\cos B \sec A)^2 = 2 \tan A (\cot B \times (\operatorname{cosec} A + \sec B))$ [3m]
- ⑤ $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$ [3m]
- ⑥ If $\tan A$ and B are acute angles, such that $\tan A = \frac{1}{2}$, and $\tan B = \frac{1}{3}$ and $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ then find $A+B$. [3m] [$A+B = 45^\circ$]
- ⑦ $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 + \tan A} = \cos A + \sin A$ [4m]
- ⑧ Evaluate: $\cot \theta \tan(90 - \theta) - \sec(90 - \theta) (\operatorname{cosec} \theta + \sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3} \tan 6^\circ \tan 16^\circ \tan 74^\circ \tan 84^\circ)$ [3m] [$\sqrt{3}$]
- ⑨ If $\cos A = \frac{12}{13}$, verify that $\sin A (1 - \tan A) = \frac{35}{156}$
- ⑩ If $\sin A + \sin^2 A = 1$, then find the value of $\cos^2 A + \cos^4 A$ [Ans: 1] [2m]
- ⑪ In ΔABC , if $\angle B = 90^\circ$, $BC = 5 \text{ cm}$ and $AC - AB = 1$, then evaluate the value of $\frac{1 + \sin C}{1 + \cos C}$ [3m] [Ans: $\frac{25}{18}$]
- ⑫ If $\sin \theta + \cos \theta = \sqrt{2} \sin(90 - \theta)$ then find the value of $\tan \theta$. [3m] [Ans: $\sqrt{2} - 1$]
- ⑬ $(\tan \theta + 2)(2 \tan \theta + 1) = 5 \tan \theta + 2 \sec^2 \theta$

(14) If $A = 15^\circ$ Prove that $4 \sin 2A (\cos 4A \sin 6A = 1)$ [2m]

(15) Prove that: $\cos \theta \sin \theta - \frac{\sin \theta \cos (90-\theta) \cos \theta}{\sec (90-\theta)} - \frac{\cos \theta \sin (90-\theta) \sin \theta}{\operatorname{cosec} (90-\theta)} = 0$ [3m]

(16) If $\sec A = x + \frac{1}{4x}$, then prove that $\sec A + \tan A = 2x$ or $\frac{1}{2x}$ [3m]

(17) $\frac{2 \cos^2 \theta - 1}{\sin \theta \cos \theta} = \cot \theta - \tan \theta$ [2m]

(18) $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$ [3m]

(19)
$$\frac{\sec (90-\theta) \operatorname{cosec} \theta - \tan (90-\theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 21^\circ \tan 63^\circ}$$
 [3m]

(20)
$$\frac{\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ}{\cot 5^\circ \cot 30^\circ \cot 35^\circ \cot 55^\circ \cot 85^\circ}$$
 [2m]

(21) If $\frac{\sin A + \cos A}{\sin A - \cos A} = \frac{5}{3}$, find the value of $\frac{7 \tan A + 2}{2 \tan A + 7}$ [3m]

(22) If $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ prove that $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ [3m]

(23) $\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta$ [3m]

(24) Evaluate: $\frac{\sec^2 (90-\theta) - \cot^2 \theta}{2 (\sin^2 45^\circ + \sin^2 45^\circ)} + \frac{2 \sin^2 30^\circ \tan^2 30^\circ \tan^2 52^\circ}{3 (\sec^2 43^\circ - \cot^2 47^\circ)}$ [2m]

(25) $\operatorname{cosec}^2 31^\circ - \tan^2 59^\circ + \frac{\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ}{3 \tan 30^\circ \tan 45^\circ \tan 60^\circ}$ [2m]

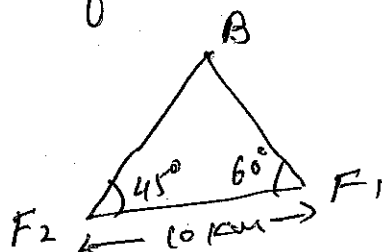
(26) $\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$

(27) $\left(\frac{1 + \sin A - \cos A}{1 + \sin A + \cos A} \right)^2 = \frac{1 - \cos A}{1 + \cos A}$ [3m]

Application of Trigonometry

① Two stations due south of a leaning tower which leans towards the north are at a distance a and b from its foot. If α and β are the elevations of the top of the tower from these stations, then prove that its inclination θ to the horizontal is given by
$$\cot \theta = \frac{b \cot \alpha - a \cot \beta}{b - a}$$

② A fire at a building B is reported on telephone to two fire stations F_1 and F_2 , 10 km apart from each other on a straight road. F_1 observes that the fire is at an angle of 60° to the road and F_2 observes that it is at an angle of 45° from it. The station F_1 sends its team.



Ans 7.32 km

(i) Why the team of station F_1 was sent?

(ii) How much distance the station F_1 team will have to travel?

③ A person standing on the bank of a river observes that the angle of elevation of the top of a building of an organisation working for conservation of wildlife, standing on the opposite bank is 60° . When he moves 40 m away from the bank, he finds the angle of elevation to be 30° .

(i) Find the length of a building and width of a river. [34.64 m]

(ii) Why do we need to conserve wildlife? [20 m]

④ At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$.

On walking 192 m towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of tower. [180 m or 307.2 m]

- ⑤ An aircraft is flying along a horizontal path PQ directly towards an observer on the ground at O and maintaining an altitude of 3000 m . When the aircraft is at P , the angle of depression is 30° and when at Q it is 60° . Find PQ .
- ⑥ A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of pedestal is 45° . Find the height of the pedestal.
- ⑦ An aeroplane flying horizontally at a height of 2500 m above the ground is observed at an elevation of 60° . If after 15 seconds, the angle of elevation is 30° , find the speed of the aeroplane in km/h .
- ⑧ A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60° . Find the total time taken by the boat to reach the shore.
- ⑨ A vertical flagstaff stands on the top of a building. The height of the flagstaff above the building is 6 m . The angles of elevation of the top and bottom of the flagstaff at a point on the level ground are 45° and 30° respectively. Find the height of the building.

Construction

- ① Construct a triangle ABC such that $AB = 5$ cm, $BC = 6$ cm and $AC = 7$ cm. Draw another triangle $AB'C'$ similar to $\triangle ABC$ such that $AB' = 3.2$ cm. [4m]
- ② Draw a line segment of length 7.6 cm and divide it in the ratio $5:8$. Measure the two parts. [3m]
- ③ Draw a triangle in which sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.
- ④ Draw a line segment PQ of length 9 cm. Taking P as centre, draw a circle of radius 5 cm and taking Q as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.
- ⑤ Draw a ^{constructed} right angle triangle similar to given triangle as per the given scale factor $\frac{2}{3}$. Also justify it.
- ⑥ Draw a circle of radius 4 cm, take a point P outside the circle. Construct the pair of tangents from this point to the circle.
- ⑦ Construct a $\triangle ABC$ in which $BC = 6.5$ cm, $AB = 4.5$ cm and $\angle ABC = 60^\circ$. Also construct a triangle similar to $\triangle ABC$, whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.
- ⑧ Draw a pair of tangents to the circle of radius 5 cm, which are inclined to each other at an angle of 60° , give steps of construction.